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Question Paper Code : 81629

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2025.

Third Semester

Civil Engineering

MA 3351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aeronautical Engineering/Aerospace Engineering/Automobile Engineering/Biomedical Engineering/Civil Engineering (Environmental Engineering)/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical Engineering (Specialised in Automobile)/Mechanical Engineering (Specialised in Smart Manufacturing)/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/Bio Technology/Biotechnology and Biochemical Engineering/Food Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Formulate the differential equation of the family of planes, the sum of whose x , y and z intercepts is one.
2. Find the particular solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = e^{4x-3y}.$$
3. The Euler Fourier co-efficient $b_n = \frac{10}{n\pi} [1 - (-1)^n]$ for the function

$$f(x) = \begin{cases} -5 & \text{for } -\pi < x < 0 \\ 5 & \text{for } 0 < x < \pi \end{cases}; f(x+2\pi) = f(x)$$
 is known. Is it possible to sum the series $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$? If so, find the sum. If not, give the reason.

4. Sketch the graph of the odd and even extensions for the function

$$f(x) = \begin{cases} x & \text{for } 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & \text{for } \frac{\pi}{2} < x < \pi \end{cases} \text{ in the interval } (-\pi, 0).$$

5. Classify the following Partial Differential Equation :

$$x^2 \frac{\partial^2 u}{\partial x^2} + (1 - y^2) \frac{\partial^2 u}{\partial y^2} = 0, -\infty < x < \infty, -1 < y < 1.$$

6. A uniform rod of length 10 cm through which heat flow is insulated at its sides. The ends are kept at zero temperature and the initial temperature at the interior points of the bar is given by $10x - x^2$, $0 < x < 10$. Write the governing differential equation with boundary conditions that associates to the heat produced.
7. State the existence condition for Fourier Transform.
8. Find the Fourier Sine Transform of $\frac{1}{5^t}$.
9. Does the Z-transform of the sequence $\{2, 3, 5, 9, 17, \dots\}$ exist? Justify your answer.
10. A car manufacturing company makes high performance cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with n cars made in the n^{th} month. Model the problem as difference equation.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Solve the partial differential equation $x^4 p^2 - yzq - z^2 = 0$. (8)
- (ii) Find the general solution of $(z^2 - 2yz - y^2)p + (xy + xz)q = xy - xz$. (8)

Or

- (b) (i) Solve : $(D^2 + 4DD' - 5D'^2)z = \sin(2x + 3y)$. (8)
- (ii) Solve : $(D^2 - DD' - 6D'^2)z = x^2 + x^2y + y^3$. (8)

12. (a) Find the Fourier series expansion of the function $f(x) = \begin{cases} 3 & \text{in } -1 < x < 0 \\ x & \text{in } 0 < x < 1 \end{cases}$, $f(x+2) = f(x)$. Hence, sum the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

Or

- (b) Find the Fourier series expansion of the function $f(x) = \begin{cases} 0 & \text{in } -\pi < x < 0 \\ \sin x & \text{in } 0 < x < \pi \end{cases}$, $f(x+2\pi) = f(x)$. Hence, sum the series $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots$

13. (a) A tightly stretched string of length 10 cm is fastened at both ends. The motion is started by displaying the string into the form of the curve $5 \sin^3 \frac{\pi x}{10}$ and then releasing from this position at time $t = 0$. Find the displacement of any point of the string at any subsequent time.

Or

- (b) Using method of separation of variables, derive all possible steady state solutions of the two dimensional heat equation in Cartesian coordinates.
14. (a) (i) Find the Fourier Transform of $f(t) = \begin{cases} 1 & \text{if } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$ and hence

compute value of $\int_0^\infty \frac{\sin x}{x} dx$. (8)

- (ii) Find the inverse Fourier Transform of $F(s) = \begin{cases} a - |s| & \text{for } |s| < a \\ 0 & \text{for } |s| > a \end{cases}$. (8)

Or

- (b) (i) Find the function for which the Fourier Transform is $\frac{5 \sin 2(s - \pi)}{(s - \pi)}$. (8)

- (ii) Using Parseval's identity for Fourier Cosine Transforms of e^{-ax} , evaluate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$. (8)

15. (a) Find the Z -transform of $\sin n\theta$, $\cos n\theta$ and find the Z -transform of $\sin \frac{n\pi}{2}$, $\cos \frac{n\pi}{2}$. Hence, use suitable property to find the Z -transform of $r^n \sin n\theta$, $r^n \cos n\theta$.

Or

- (b) Solve the difference equation $u_{n+2} + 2u_{n+1} + u_n = n$, $u_0 = 0$, $u_1 = 0$ using Z -transform.